Discrete–Time Flux Observer for PWM Inverter Fed Induction Motors

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Abstract. For the purpose of flux oriented control a standard observer structure is applied to get the unmeasurable rotor flux of an induction motor. Main emphasis is put on discrete–time considerations with respect to microprocessor realizations. This aspect concerns the internal observer structure as well as the data acquisition and sampling strategy, which is synchronized with the pulse width modulation.

Measurement results show that the proposed observer provides good results for both motor and generator operation modes and even at standstill. Moreover, the inherent parameter robustness of the used structure is demonstrated.

Keywords. Flux observer, induction motor, discrete–time realization, sampling strategy.

INTRODUCTION

Knowledge of the rotor flux linkage of an induction motor is necessary to realize strategies of flux oriented control. Because the rotor flux cannot be measured directly, observer structures are used, which calculate an estimated rotor flux from measurable quantities such as voltage, current and rotor speed.

Usually flux calculators are based on particular models of the induction motor. Frequently used are the so–called voltage and current models, of which numerous variations exist (cf. Nuß, Huck, 1990). The stator model gives good results for higher rotor speeds but it is not applicable for lower speeds or at standstill due to open–loop integrators in this structure. The rotor model is usable for all rotor speeds but it depends on the temperature–varying rotor resistance. Recently arrangements of two flux calculators have been presented to reduce the disadvantages of each particular structure (eg. Bauer, Heining, 1989).

Here an observer structure is used, which is well known from control and filtering theory since the early sixties (Kalman, 1960, Luenberger, 1963), but which is seldom applied to problems of drive systems (but Zägelein, 1984). This structure has the advantage of affecting the estimation error with respect to its dynamics or parameter sensitivity independently of the system dynamics by an appropriate choice of feedback coefficients.

Regardless of the chosen observer structure, most approaches retain a continuous–time point of view. The aspect of the implementation on sampling digital microprocessor systems is often neglected. The desired continuous–time behavior is approximated by using high sampling rates. With these quasi–continuous approaches integrations are replaced by stepwise summations, the numerical performance of which may be crucial.

A discrete–time approach does not only save computing power, moreover it can operate hand in hand with a discrete–time controller and a pulse pattern generator. A PWM inverter can only influence the voltage twice during a pulse period (once each pulse edge). A voltage modulating method such as regular sampling or space vector modulation takes this into account and sets up sampling periods by itself. Thus it is very natural to synchronize controller and observer with the discrete–time operation mode of the pulse pattern generator in order to achieve optimal performance of the complete drive system.

A special problem to be solved related to such a discrete–time observer is the data acquisition of current and voltage, which have high harmonic contents. The paper shows how it is possible to suppress the harmonics without analog filtering by adequate sampling strategies.

CONTINUOUS–TIME OBSERVER

A standard observer structure for a linear system is shown in fig. 1. The induction motor (the process to be observed) is described by a system of differential equations in matrix notation

$$\dot{\psi}(t) = A(\omega_{rs}(t))\,\psi(t) + B\,u_s(t) \tag{1}$$

$$i_s(t) = C\,\psi(t). \tag{2}$$

The stator voltage $u_s(t)$ and the stator current $i_s(t)$ appear as measurable input and output variables while the state vector

$$\psi(t) = \begin{bmatrix} \psi_s(t) \\ \psi_r(t) \end{bmatrix} \tag{3}$$

is unknown. The state vector consists of stator and rotor flux linkages. All these quantities are complex values. The real and the imaginary parts refer to the two orthogonal stator–fixed axes of the motor. The continuous–time system matrix A, which depends on the angular velocity of the rotor ω_{rs}, and the input and output matrices B and C are derived from the basic equations as

$$A(\omega_{rs}) = \begin{bmatrix} -\dfrac{R_s}{\sigma L_s} & \dfrac{R_s M}{\sigma L_s L_r} \\ \dfrac{R_r M}{\sigma L_s L_r} & -\dfrac{R_r}{\sigma L_r} + j\omega_{rs} \end{bmatrix} \quad (4)$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (5)$$

$$C = \begin{bmatrix} \dfrac{1}{\sigma L_s} & -\dfrac{M}{\sigma L_s L_r} \end{bmatrix} \quad (6)$$

The observer shown in fig. 1 consists of a model of the induction motor, which is fed by the same input $u_s(t)$ while the output error $\hat{i}_s(t) - i_s(t)$ is used for an error correction via the feedback matrix L. Algorithms for designing this feedback are well known from control theory, eg. the pole assignment or the optimal observer design (cf. Kwakernaak, Sivan, 1972). The observer provides an estimation $\hat{\psi}(t)$ of the internal state vector $\psi(t)$.

This observer is continuous in time as the original process. Therefore it cannot be realized in this form on a digital microprocessor system. It is necessary to change over to discrete-time considerations.

DISCRETE–TIME OBSERVER

The input function $u_s(t)$ and the rotor speed $\omega_{rs}(t)$ are assumed to be constant during a sampling period $(k\tau, (k+1)\tau)$. This is a satisfactory approximation concerning $\omega_{rs}(t)$, because the rotor speed does not vary very fast. The assumption regarding the stator voltage $u_s(t)$ will be discussed later. With this premise the discrete-time observer (fig. 2) is described by the difference equations

$$\hat{\psi}((k+1)\tau) = \Phi(\omega_{rs}(k\tau))\, \hat{\psi}(k\tau) + H\, u_s(k\tau) + K\left(i_s(k\tau) - \hat{i}_s(k\tau)\right), \quad (7)$$

$$\hat{i}_s(k\tau) = C\, \hat{\psi}(k\tau) \,. \quad (8)$$

As it is well known, the transition matrix Φ and the discrete-time input matrix H are evaluated from the continuous-time matrices A and B via

$$\Phi(\omega_{rs}) = \exp\left(\tau\, A(\omega_{rs})\right), \quad (9)$$

$$H(\omega_{rs}) = \int_0^{\tau} \exp\left(t\, A(\omega_{rs})\right) \mathrm{d}t\, B \,. \quad (10)$$

Although this is the exact solution of the discretization problem, the expensive computational effort for evaluating Φ and H prohibits their on-line calculation. Unfortunately it is not possible to carry out these calculations off-line before starting the observer since the system matrix A and therefore Φ and H depend on the varying rotor speed ω_{rs} as a parameter. Hence, for quick on-line calculations approximation methods are used. For this the integration algorithm of Euler is mostly applied (eg. Atkinson et al., 1989), which is characteristic of quasi-continuous approaches. This means, Φ and H are replaced by their first-order approximations

$$\Phi(\omega_{rs}) \approx I + \tau\, A(\omega_{rs}) \,, \quad (11)$$

$$H \approx \tau\, B \,, \quad (12)$$

where I denotes the identity matrix. As pointed out (Böcker, 1991), the application of this algorithm to the system of differential equations (1) of the induction motor may give rather poor results, particularly for parameters of motors with

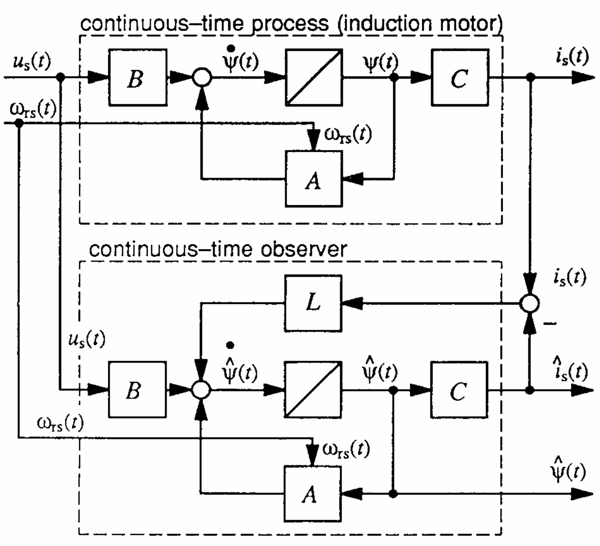

Fig. 1 Structure of a Continuous–Time Flux Observer

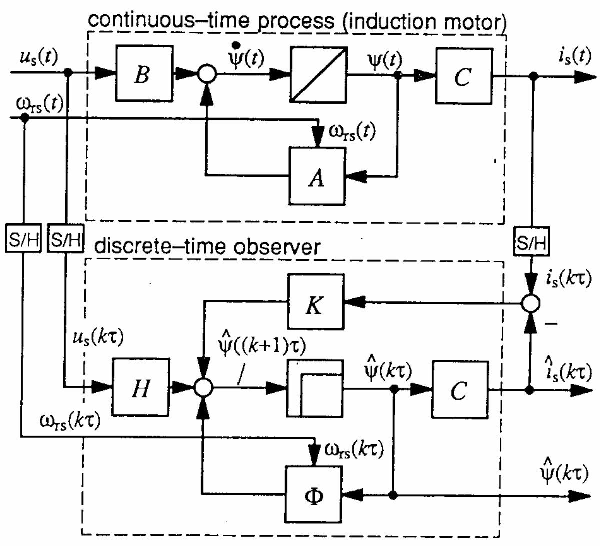

Fig. 2 Structure of a Discrete–Time Flux Observer

large time constants. Even stability problems may occur. The reason for this is the crucial numerical performance of Euler's algorithm for weakly–damped systems. Thus Hodapp (1989) uses the Adams–Bashforth algorithm, which is of second order, while Pfaff and Segerer (1989) propose series expansions of the transition and input matrices of order three. One of the authors proposed a modified approximation, which is of first order with the exception of a rotational transform $\exp(j\tau\omega_{rs})$ (Böcker, 1991). The proposal gives good accuracy while the numerical expenditure is small. It should be applied also in the present context:

$$\Phi(\omega_{rs}) \approx \begin{bmatrix} 1-\tau\dfrac{R_s}{\sigma L_s} & \tau\dfrac{R_s M}{\sigma L_s L_r} \\ e^{j\tau\omega_{rs}(k\tau)}\,\tau\dfrac{R_r M}{\sigma L_s L_r} & e^{j\tau\omega_{rs}(k\tau)}\left(1-\tau\dfrac{R_r}{\sigma L_r}\right) \end{bmatrix}, \quad (13)$$

$$H \approx \begin{bmatrix} \tau \\ 0 \end{bmatrix}. \quad (14)$$

SAMPLING STRATEGY AND DATA ACQUISITION

As pointed out in the introduction, the sampling should be synchronized with the pulse generator. Using the regular sampling modulation method as depicted in fig. 3 (or similar the space vector modulation), the sampling instants $t = k\tau$ of the observer should be equal to those of the modulation.

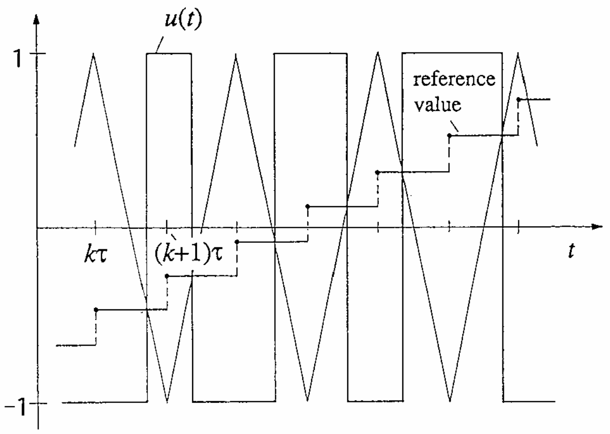

Fig.3: Regular sampling pulse width modulation

Voltage data acquisition

In the preceding section the input $u_s(t)$ (the stator voltage) was assumed to be constant during a sampling period. This assumption is obviously not true because of the pulsating voltage. However, a voltage pulse causes a current ripple, but the current fundamental is influenced by the mean value of the voltage pulse only due to the smoothing behavior of the leakage inductance. (Just this is the justification for applying a pulse width modulated inverter instead of a power amplifier to feed the motor.) If only the dynamics of the fundamental should be considered, it is permitted to substitute the continuous–time behavior of the voltage $u_s(t)$ in each sampling period $(k\tau, (k+1)\tau)$ by its mean value. This value can be used as $u_s(k\tau)$ to feed the discrete–time observer.

The voltage mean value of one sampling period is ideally equal to the voltage reference value of the pulse pattern generator (cf. fig. 3). In reality differences are caused by inverter losses and switching delay times. For applications of low performance nevertheless, the voltage reference value of the modulator can be used instead of a measured mean value.

In this paper a data acquisition method is proposed, which directly supplies the voltage mean value of a sampling period. For that purpose the stator voltage is measured by an arrangement of a voltage controlled oscillator and a digital counter. The difference of the counter contents between the beginning and the end of a sampling period immediately provides a digitalized data of the voltage mean value. In fact, this is a special realization of an A/D converter. It can be seen from fig. 4 that it is possible to detect the fundamental oscillation of a pulsating voltage by this method without any analog filtering.

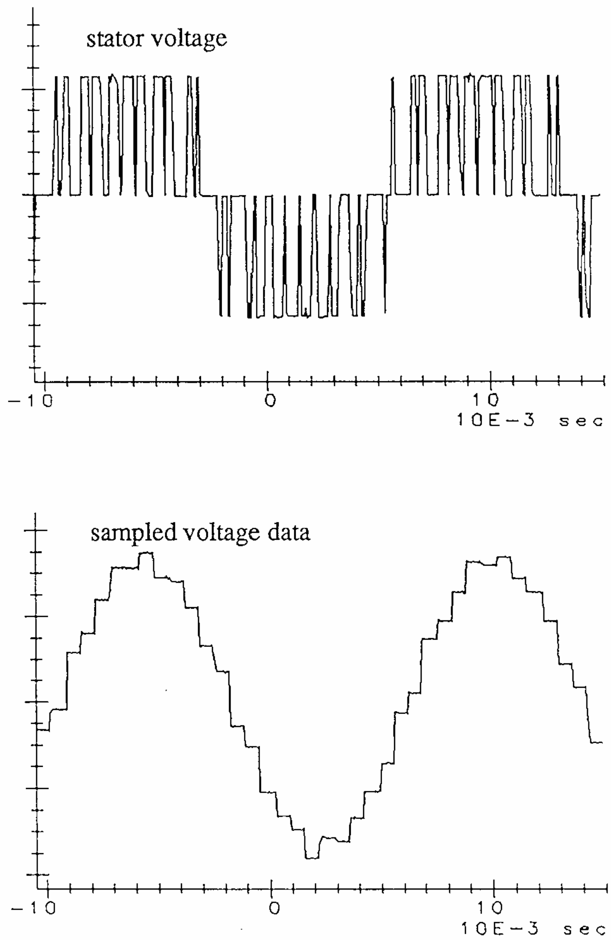

Fig.4: Measurement results with proposed voltage data acquisition method

Current data acquisition

At instants $t = k\tau$ the current is sampled, held and digitalized by an A/D converter. The measurement results shown in fig. 5 demonstrate that is possible to suppress the ripple content of the motor current by this sampling strategy. Similar to the voltage sampling strategy no analog filtering is needed to extract the fundamental (cf. the sampling strategy applied to a four–quadrant converter by Endrikat, 1991).

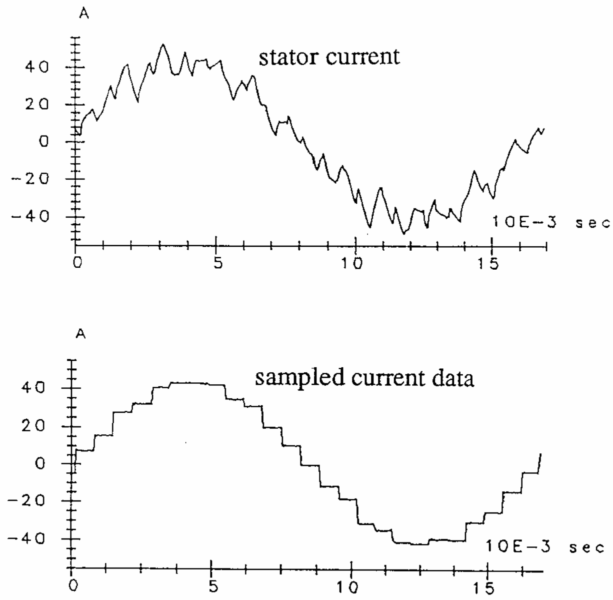

Fig.5: Measurement results with proposed current data acquisition method

FEEDBACK DESIGN

Methods such as pole assignment or optimal observer design are available for design the feedback K of discrete–time observers similar as for continuous–time ones.

At first, the recursive optimal observer algorithm (known as Kalman filter, cf. Kwakernaak, Sivan, 1972) was investigated. This algorithm is quite simple and can be carried out on–line on a microprocessor. Its advantage is its self–adaptability to varying rotor speeds and to varying other motor parameters in view of automatic parameter identification. The design is optimal with respect to a linear functional of the estimation error covariances, whereby white noise disturbances are assumed. This assumption is usually not fulfilled, so that this design may not give an appropriate observer. First experiments had shown that the behavior regarding the estimated torque (cf. eq. (16)) is unsatisfactory. Therefore a manually designed feedback

$$K = \begin{bmatrix} K_s \\ K_r \end{bmatrix} \qquad (15)$$

is used. The real–valued elements K_s and K_r vary with the rotor speed ω_{rs}.

REALIZATION AND PRACTICAL RESULTS

The observer algorithm was implemented on a TMS 320 signal processor. The inverter operates with a pulse frequency of about 900 Hz. Regarding the sampling strategy this yields a sampling frequency of 1800 Hz and a sampling period of 555 µs. In each period one cycle of the observer algorithm is processed.

The measurement of the true flux demands great effort. Hence, the quality of this observer is assessed by comparing the measured torque T with the torque calculated from estimated flux and current by (* denotes the complex conjugate value)

$$\hat{T}(k\tau) = 1.5\, n_p M/L_r \, \mathrm{Im}\left(\hat{i}_s(k\tau)\, \hat{\psi}_r^*(k\tau)\right) . \qquad (16)$$

Measurement results are shown in figures 6 to 8 for various rotor speeds in steady state. The estimation error $\hat{T} - T$ is plotted versus the measured torque T (positive signs of T indicate motor, negative signs generator operation mode,

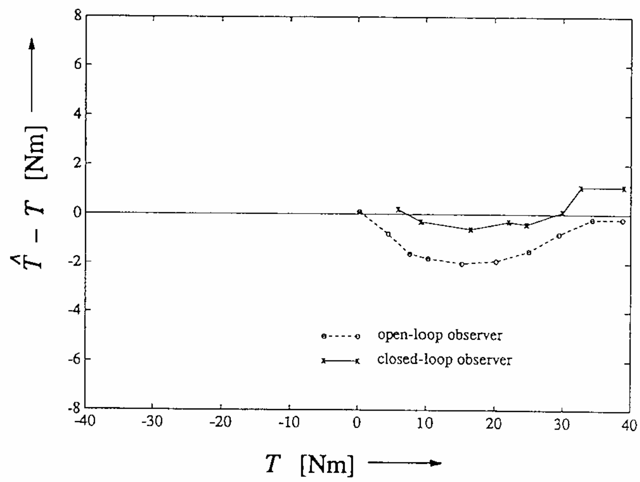

Fig.6: Torque estimation error versus measured torque at standstill

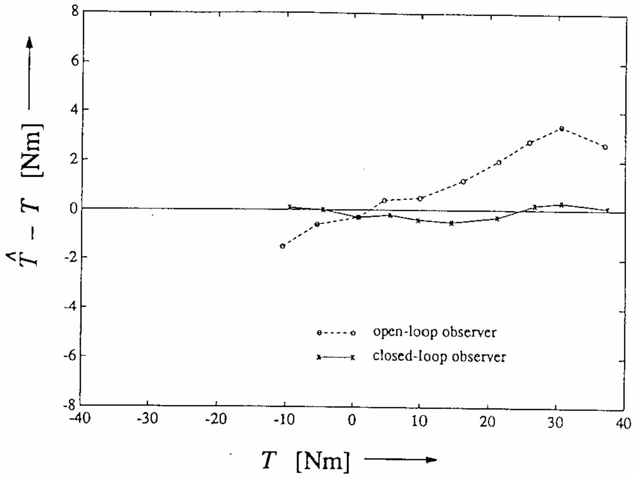

Fig.7: Torque estimation error versus measured torque at $\omega_{rs}/2\pi = 3$ Hz

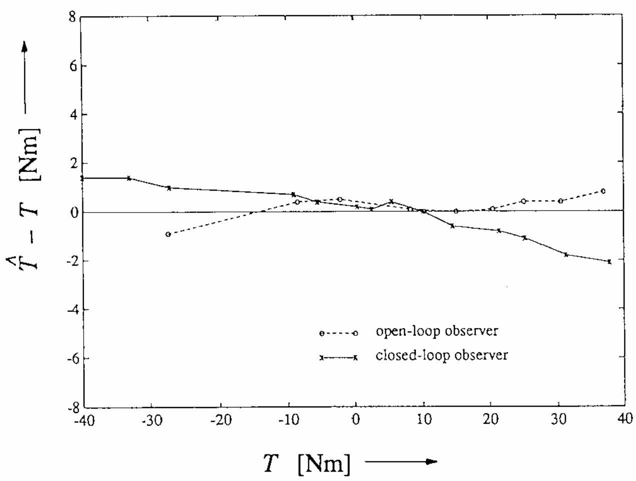

Fig.8: Torque estimation error versus measured torque at $\omega_{rs}/2\pi = 15$ Hz

respectively). The observer was tested in an open–loop mode with $K = 0$ (dashed lines) and with closed feedback (solid lines). Even the open–loop structure gives good results, because the observer parameters were adjusted carefully. Closing the observer feedback, the error can be reduced furthermore. Results at standstill are as good as for higher rotor speeds. The torque estimation error is typically smaller than 2 Nm, which is about 3 % of the rated torque.

PARAMETER SENSITIVITY

An observer should be robust with respect to parameter variations. Therefore the observer was tested by increasing its parameter of the mutual inductance M by 50 %. While the open–loop structure gives large errors (fig. 9), the feedback compensates for detuning almost completely.

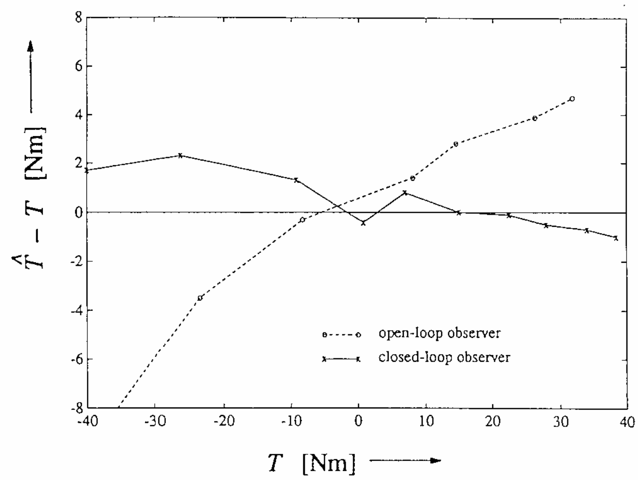

Fig.9: Torque estimation error versus measured torque at $\omega_{rs}/2\pi = 15$ Hz with an observer parameter of the mutual inductance M increased by 50%

As a second test the observer parameter of the rotor resistance R_r was increased by 20 %. Results at standstill can be seen in fig. 10. Without feedback the relative error of the torque estimation is also about 20 %. Using the feedback, the torque error can be decreased by half.

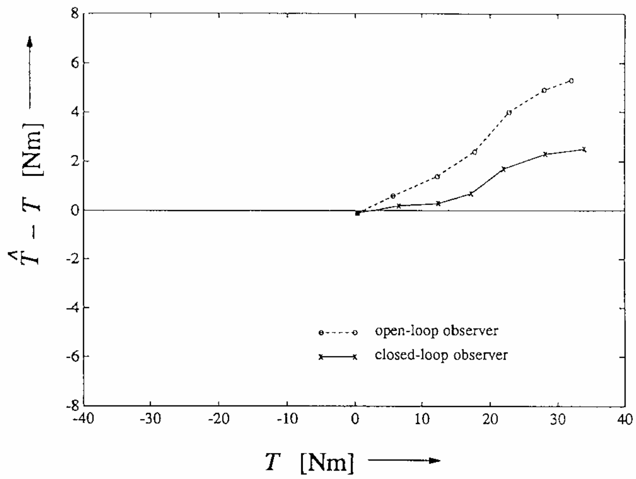

Fig.10: Torque estimation error versus measured torque at standstill with an observer parameter of the rotor resistance R_r increased by 20%

CONCLUSIONS

A flux observer has been proposed, which is based on a standard observer structure. Emphasis was put on discrete–time considerations with respect to microprocessor implementations. For this, a discrete–time model of the induction motor was used, the computational effort of which is low. Sampling strategies and data acquisition methods, which fit the discrete–time mode of the observer, were presented. With these methods fundamentals of voltage and current can be detected without any analog filtering.

Because the true flux was not measured, the estimated torque was used for an assessment. It was demonstrated that the proposed observer gives good results for the complete speed range, even at standstill. The observer is robust with respect to parameter variations.

The observer will be integrated into a discrete–time controller structure. Thus optimal performance of the complete drive system can be achieved due to homogeneous discrete–time considerations of observer and controller algorithms, pulse width modulation and data acquisition.

LIST OF SYMBOLS

A	continuous–time system matrix
B	continuous–time input matrix
C	output matrix
H	discrete–time input matrix
I	identity matrix
Im	imaginary part
i_s	stator current
j	imaginary unit
K	feedback matrix of discrete–time observer
k	sampling index
L	feedback matrix of continuous–time observer
L_s, L_r	stator and rotor inductances
M	mutual inductance
n_p	number of pole pairs
R_s, R_r	stator and rotor resistances
T	torque
t	time
u_s	stator voltage
σ	leakage coefficient
τ	sampling period
Φ	transition matrix
ψ	state vector
ψ_s, ψ_r	stator and rotor fluxes
ω_{rs}	angular velocity of the rotor (rotor speed)

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